Effect of Critical Thinking Skills on Misconceptions of Students in Learning Mathematics at the Elementary School Level

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Abstract

The aim of mathematics education in schools is to create logical reasoning and critical thinking ability in students. These skills development not only help students understand better but can also help remove the misconceptions they have about mathematical concepts. The purpose of this study was to investigate how critical thinking skills can help address misconceptions in mathematics learning at the elementary level. The research used an A-B-A design to determine the effects of an intervention, which is a perfect design for this purpose. 35 eighth-grade students who were enrolled in the current academic year were participants in this study. Three researcher-developed tools facilitated data collection. The first and initial tool was called the Test of Critical Thinking Skills (ToCTS), which measured students' critical thinking math skills. The second tool, the Test of Misconceptions (ToM), was developed to identify specifically how the students thought they were solving the problems. Finally, an interview protocol was used to learn about the students' reasoning regarding their answers on the tests. The mathematics curriculum of grades three to eight formed the basis of the development of these instruments, both close-ended and open-ended questions. Descriptive and inferential statistical methods were used in data analysis, and qualitative content from the interviews was applied to thematic analysis. A poor level of critical thinking skills in factoring equations and selecting proper solving methods for linear equations was shown by students. In addition, students had difficulty solving complete mathematical problems with linear equations. A significant gap was identified in their understanding and application of the BODMAS rule during algebraic operations. Based on these findings, the study recommended fostering critical thinking skills to improve students' abilities in factoring equations. Teachers should provide clear and comprehensive guidance to help students solve mathematical problems effectively. To address misconceptions, educators could introduce simplified problem-solving methods and reinforce students' understanding of fundamental operations and rules. Specifically, it is essential to provide targeted instruction on the application of the BODMAS rule in algebraic contexts to enhance students' problem-solving capabilities.

Keywords*: SDGs, Remittances, Poverty Alleviation, Panel ARDL, Developing Countries*

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Introduction

Critical thinking is reasoning out of conclusion and evaluation of information with proofs only before selecting or discarding the facts present. As such, it encourages the students to think, question issues, make answers for issues, and make academic decisions when confronted with challenges (Akram, Javed, Niaz, & Mahmood, 2022; Khan, Akram, Shams, & Mahmood, 2023). Neither is it true that CTS directly relates to mental abilities or higher-order thinking skills such that one can either know about one's own knowing and about the knowing of others or not know (Kuhn, 1999). Critical thinking also includes reflective thinking and an impression that the information they acquired rather than simply accepting different concepts without investigating (Fahim el al., 2012). Students refuse to accept an idea if they don't get experience in a systematic and coherent process to get the background reality. Teaching critical thinking skills to students requires the promotion of thinking skills. Therefore, students with the tendency to have critical thinking skills should be able to search for seemingly related ideas, create and evaluate arguments, identify common practices of bad thinking, and tackle problems systematically (Mahmood, Akram, et al., 2024).

Advanced-level mathematical problems, which can develop students' critical thinking ability, include thinking, synthesis, and evaluation (Akram et al., 2022). It is overwhelmingly obvious that there is a connection between critical thinking skills and mathematics. Critical thinking skills are found to be positively encouraging student's abilities in solving mathematical problems, and many researches have shed light on it. The cycle of basic inference in Mathematics means bringing together information in science, numerical thinking, and numerical confirmations to numerical misjudgment (Aizikovitsh & Amit, 2010; Rajendran, 2010). Teachers at the elementary school level play a vital part in enhancing students' critical thinking skills through positive school climate (Mahmood, Ismail, & Omar, 2024). Perception regarding removing misconceptions with the help of critical thinking skills is worthwhile (Niaz, Akram, Mushtaq, & Mahmood, 2023).

Literature Review

Algebraic misconceptions can be categorized into three main areas of algebra at elementary school level: (1) Algebraic expressions, (2) Polynomials and (3) Linear equations. In algebraic expressions students have misconceptions regarding: 1) Using symbols as variable 2) Understanding algebraic expressions as algebraic equations 3) Use of variables and understanding how different variables have different meanings (Mulungye, 2016). Campbel (2009) noted that students have misconceptions in simplification of rationale expressions. Chow (2011) observed that students misinterpret the procedures used in solving linear equations. Li (2006) describes the conceptual misconception of using variable as place holder. They think letters represent different numbers. Boot and Koediger (2008) describe procedural and conceptual misconceptions in elementary level algebra. Procedural misconceptions are adding like terms, incorrect application of inverse operations. Conceptual misconceptions include using signs and operations. Mulungye et al., (2016) observed the misconceptions in distributive law, incorrect use of signs and difficulty in using distributive property. The aim of developing Critical Thinking Skills (CTS) is the correctness of thought and clarity of concepts, which is required to bring conceptual change and avoid misinterpretations of the problem (McGregor, 2007).

Ennis (Sabandar, 2019) gave set of six skills to teach content based critical thinking skills

1. Focus on Identification and interpretation of the main problem is that explain the main problem in ones' own words and deciding what to find out? What will be the output of the problem? How the problem will be proved and what will be the complete solution.
2. Analyzing the main problem is about making arguments that support the conclusions. It is also about finding details and proofs that can support reasons of making inferences. It is about the detailed examination of ideas.
3. Inference is the proposed reasons to prove the question. Further a student decides either the reasons are true or not?
4. Evaluation of the evidence and arguments is the ability to evaluate proposed arguments. This ability of evaluation of the arguments can enhance students' ability of thinking correctly and making inferences.
5. Self-Regulation is the stage searching for truth and correctness of thought. At this stage students will be able to avoid misconceptions and think correctly.
6. Overview is the process of overall understanding the problem. This process includes above all steps to be review. After that students are able to avoid faulty preconceptions and build their own thinking with true reasons.

In addition to this development, critical thinking skills supporting mathematical problem solving is also supported by the process of developing critical thinking according to Facione (1990) and Ennis (1987). In Psychology, Kowalski & Taylor (2004) found that ability and critical thinking skills are predictors of getting students to remove their misconceptions. Generally, in conventional methods teachers do not stress on valid arguments behind the answer. Mostly, in conventional methods of teaching a correct answer is considered as the sign of success (Haron et al., 2024). In doing this reasoning skills are needed. To extract conclusions and logical arguments behind the solution of a problem is necessary for elementary level students. However, Researchers found that in mathematics class this ability of reasoning seems to vanish. To aware the students that they already have this ability of reasoning out of school life is necessary. By doing that students are able to readily transform the information given to them. It becomes easy for teacher to engage their students in given tasks (Vosniadou, 2021).

Statement of the Problem

This study aimed to explore and address misconceptions in algebra among elementary school students. The research highlighted the significance of fostering a comfortable learning environment for students and emphasized the role of critical thinking skills in correcting these misconceptions. Developing advanced logical reasoning and critical thinking abilities is considered essential for helping students overcome misunderstandings in mathematics (Ferrett, 2002). According to Applebaum (2010), mathematics teachers play a crucial role in enhancing their students' critical thinking capabilities to clarify concepts and dispel misconceptions. In the context of mathematics, critical thinking serves as a foundational tool for concept clarity and addressing errors in understanding. However, there is very little research conducted on the application of critical thinking skills to help students to understand their misconceptions about mathematics in Pakistan. Experimental studies are still needed desperately to explore and fully address these misunderstandings. Given this research gap, this study attempts to fill it by trying to improve students' critical thinking skills with appropriate strategies and, in doing so, correct their mistakes in algebra.

Research Objectives

The objectives of the study were as follows:

1) To evaluate the effectiveness of pedagogical interventions that is based in critical thinking skills in reducing the difficulties that students encounter in dealing with their misconceptions in mathematics.
2) To study the effect of critical thinking-based teaching strategies on students' understanding of the concept of polynomials and related misconceptions.
3) To evaluate how the use of critical thinking-oriented instructional methods influences students' misconceptions about the concept of factorization in mathematics.
4) To examine the role of critical thinking-oriented pedagogical approaches in highlighting events of misconceptions about linear equations in students.

Conceptual Framework of the Study

The critical thinking skills are in close ties with misconceptions in mathematics. Obviously, there is a widespread recognition among educational experts that students need better skills in logic and creativity—skills that will serve them well when their schooling is over. Moreover, although this emphasis was taken for granted, overall, the performance in mathematics of the students has never been suboptimal — not only in terms of their grades, but also in their capacity to understand and apply mathematical concepts in the practical, real world contexts. In essence, many students struggle to translate their classroom learning into meaningful, everyday problem-solving.

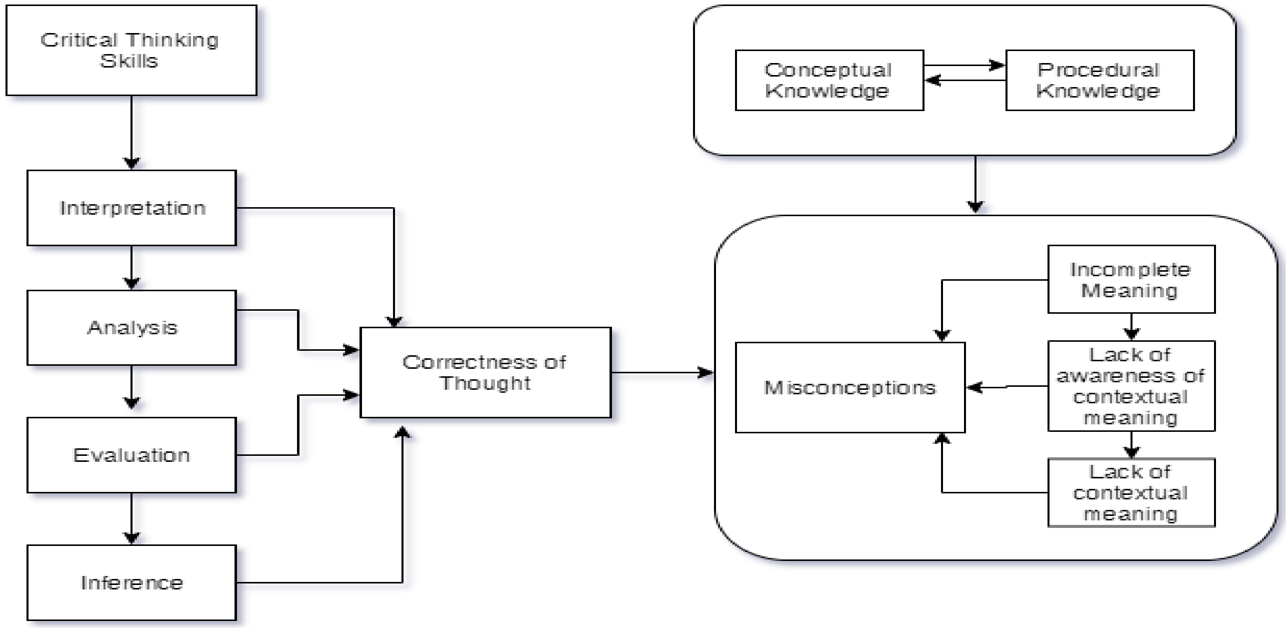

Figure 1: ***Pathways to develop Critical Thinking in Mathematics***

Methods and Procedures

The purpose of the study was to find how critical thinking skills influenced students' misconceptions of learning mathematics. In this study, two variables, CTS and Misconceptions of mathematics, were used. A single-subject research design using an A-B-A withdrawal design was conducted in this study. A single subject may be a class of people, a group of people, a population who have the same problems to be treated, etc. This design aims to measure subjects' behavior as a dependent variable over time and assess improvement in behavior in order to compare to improvement in behavior as a way to determine if treatment is successful (Yao et al., 2013).

Table 1: ***A-B-A Experimental Design***

Baseline A	Treatment B	Baseline A
O-O-O-O-O	X-X-X-X-X	O-O-O-O-O

O = Observations

X = Treatment

Sample Lesson Plan

Concept: Solving Linear Equation in One and Two Variable

Grade: 8th Time duration: 35 Minutes

Previous Knowledge Questions:

What is constant?

What is Variable?

Instructional Material:

Table 2:

Step 1 **Discussion on previous knowledge questions before starting the lesson**		
Learning Targets	Teacher Activities	Student Activities
Getting ready students for Lesson	1-Before we start, I want to ask you one interesting question. Have you ever learned "Variable and Constant" when you were a very small kid? 2-But I say yes, you have learned about variables and constants since childhood. Do you know where your name came from? When you were unnamed your mother wants to call you. She gave you a name. 3-Similarly in mathematics when we represent numbers 2, 3, and 4 with some other names like a, b, and c. What will we call them? Today we will read about the Linear Equations and Their solutions. The equation in which the variable is raised to the first power and is not in a denominator, under a square root, or inside an absolute value bar is called a linear Equation.	1-Simple flat answer by students can be NO. 2-Students will ask how. 3-Students will understand probably and answer yes we know these are called variables and constants. Our name is a variable that we learned when we were kids.
Step 1 **Identification and Interpretation of Information**		
To develop skills in comprehending and decoding the information given in the problem	Here, the teacher will write the problem on the board. Write an equation and solve it. Yusra finds Rs. 10 on the way to your home and puts them in your bag. Later, when you check your pocket, you find that you have Rs.28. How much money did you have before you found Rs. 10? The teacher will ask the students to share their points of view on it. The teacher will guide the students in reading the	Here students will note down the problem on their notebooks

	question carefully and finding what is given in the word problem. What is to be found out about the given problem? Read the problem and decide what "x" represents. Define all other variables described in the problem in terms of x. Write down the equation Now solve the equation	
Step 2 Information Analysis		
To develop skills for detailed examination of information, questions, statements, and concepts.	To examine the problem in more detail, the teacher will ask some questions. Decide the steps involved in solving the problem.	Students will try to perform these steps guided by the teacher 1) Distribute (eliminate all parenthesis) 2). Combine all 'like' terms on each side. 3). Using + and, move all variable terms to one side of your equation and all numbers to the other side. 4). Using * and /, "get rid" of any coefficient of your variable besides 1
Step 3 Evaluation of the argument		
To develop skills in evaluating or judging the integrity or value of statements.	Now teacher will give them an example and ask them to solve the equation by using the steps they earlier decided and then check the answer	Students will solve the example with the same procedure that the learn from the teacher and also answer the questions asked by the teacher. Students will also evaluate the arguments they built for the solution of the problem and test these arguments by solving the problem.
Step 4 Inference		
To develop skills in drawing reasonable conclusions from data, statements,	Here, the teacher will ask the students whether they make correct decisions or not. If students solve wrong question. Make their own judgments.	Students will give feedback that they answer correctly.

judgments, questions, and concepts.		
Step 5 Self-regulation		
To develop the skills that will allow students to monitor their cognitive activities with a sight to correcting themselves.	The teacher assists students in applying skills in reassessing their judgment through (i) self-examination and (ii) self–correction.	The students were able to reassess their work by cross-checking the procedures and the answers they got.

Validity and reliability

Validity and reliability were tested for the study's instrument to ensure accurate measurement. Four types of rationality—content, predictive, concurrent, and construct—were considered, with validity defined as measuring what is intended (Remmer, 1965; Ghauri & Gronhaug, 2005). A pilot test involving 60 grade VIII students from public schools confirmed the format and content understanding. Confirmatory Factor Analysis (CFA) further established validity, while reliability was confirmed with a Cronbach's Alpha above 0.7. Multicollinearity was reflected by broad-mindedness and variance inflation factor (VIF) values, with VIF kept under 3.3 for acceptable measure accuracy.

Table 3. ***Baseline (A) Data Collection Schedule for Post-10th week***

Session1	**10^{th}Week**
Monday – 21 November, 2020 to 26 November, 2020	
Test Conduction 1	AICT (Assessment Instrument for Critical Thinking) is based on the concept of Polynomials
Test Conduction 2	AICT (Assessment Instrument for Critical Thinking) Factorization
Test Conduction 3	AICT (Assessment Instrument for Critical Thinking) Linear Equations
Test Conduction 4	Post-Test Misconceptions

Data Analysis

Identification of the misconceptions of the students in learning Mathematics at the elementary school level

Table 4: ***Completing and analyzing a repeating pattern***

Reasons	Frequency	Percentage
Choose a nearby number of the given option	5	14.3
Misconception of adding rule	22	62.9
Right answer	3	8.6
Misconception in adding	5	14.3

Table 4 highlights the misconceptions students exhibited regarding completing and analyzing repeating patterns. The data indicates that a majority of students (62.9%) struggled with misconceptions related to adding rules, while 14.3% selected numbers that were close to the given number, further reflecting their misunderstanding of adding rules. Only a small percentage of students (8.6%) managed to choose the correct answer, demonstrating a clear difficulty in grasping the concept. Overall, the findings reveal that misconceptions about adding rules were the most prevalent, with significantly fewer students providing correct responses compared to those falling into other categories of misunderstanding.

Levels of Self-Regulation Skills in Mathematics Learning

Level 1: Searching

At this level, students primarily focus on identifying and gathering relevant information needed to approach a mathematical problem. They rely on external resources or guidance to understand the problem and often require support to formulate a plan for solving it.

Level 2: Self-Correction

Students at this stage begin to critically evaluate their own work. They are able to identify errors or gaps in their approach and make necessary adjustments independently. This level demonstrates an improvement in self-monitoring and an understanding of mathematical concepts.

Level 3: Complete Solution

At the highest level, students demonstrate the ability to solve mathematical problems independently and accurately. They not only apply appropriate methods and strategies but also validate their solutions with confidence, showcasing full mastery of self-regulation in mathematics learning.

Table 5:

Concept	Skill Level	Number of Students	Percentage (%)
Polynomials	Basic (Level 1)	17	52.9
	Intermediate (Level 2)	16	45.7
	Advanced (Level 3)	2	5.7
Factorization	Basic (Level 1)	20	57.1
	Intermediate (Level 2)	15	42.9
	Advanced (Level 3)	0	0
Linear Equations	Basic (Level 1)	18	51.4
	Intermediate (Level 2)	16	48.6
	Advanced (Level 3)	1	2.8
Overall	Basic (Level 1)	18	51.42
	Intermediate (Level 2)	16	45.71
	Advanced (Level 3)	1	2.85

The data in Table 5 highlights the self-regulation skills exhibited by students in critical thinking within the context of mathematics. These skills were evaluated across three mathematical concepts: polynomials, factorization, and linear equations (52%, 57.1%, and 51%, respectively). A majority of students demonstrated proficiency at the basic level, where they could search for and gather information related to the problem. Approximately 45% of students exhibited the ability to self-correct their approaches when solving problems.

In polynomials, most students were found to operate at Levels 1 and 2, while only a small proportion successfully reached the highest level, demonstrating the ability to fully solve the problems. Similarly, in factorization and linear equations, very few students progressed to the final stage, which required providing a complete and accurate solution. Overall, the majority of students were observed to remain at the initial searching stage of critical thinking, with only a small percentage achieving the highest level of problem-solving ability.

Step 3: Development of Critical Thinking Skills in Mathematics Learning

Table 6: Impact of CTBI on Students' Critical Thinking Skills Across the Three Phases of Intervention

The next phase of study was to evaluate the development of critical thinking skills of students at three different stages of intervention. Results demonstrated how student use of the Critical Thinking Based Intervention (CTBI) affected their approach and solving of mathematical problems. The systematical assessment of each phase was made to see the extent of their impact in improving students' critical thinking capability; findings indicated gradual progress in problem-solving and self-regulation skills.

The structured approach also resulted in measurable outcomes, demonstrating that well-targeted interventions can effectively help students think critically in mathematics.

The Effect of (CTBI) on Students' Critical Thinking Skills During Three Phases of Intervention

Test Phase	Mathematical Concept	Mean (M)	Standard Deviation (SD)	Degrees of Freedom (df)	F-Value	Wilks' Lambda	p-Value	Partial Eta Squared
Baseline	Polynomials	22.23	1.43	8, 272	584.79	0.01	0	0.945
	Factorization	21.43	1.63					
	Linear Equations	22.8	1.32					
Intervention	Polynomials	31.8	1.23					
	Factorization	32.43	1.24					
	Linear Equations	32.51	1.17					
Withdrawal	Polynomials	30	0					
	Factorization	29.46	0.67					
	Linear Equations	29.91	0.28					

***$p < .001$

Table 4.3.1 shows the differences between ATCTS scores for students in different stages of the study for three areas of mathematics. In order to see the effect of the teaching strategy on students' critical thinking skills, a one-way repeated measures ANOVA was conducted on these variables. The mean CTStest scores demonstrated significant variation over the ten weeks: **$F(8, 272) = 584.788$, $p = .000$, $\eta p^2 = .945$**. The mean pair-wise scores of observations 4, 5, 6, 7, and 8 had significant differences from each other using Bonferroni post hoc test. The results indicate that the intervention substantially influenced student performance in the three phases of the experiment. Moreover, the partial eta square also shows that treatment has a substantial effect on improving students' critical thinking skills.

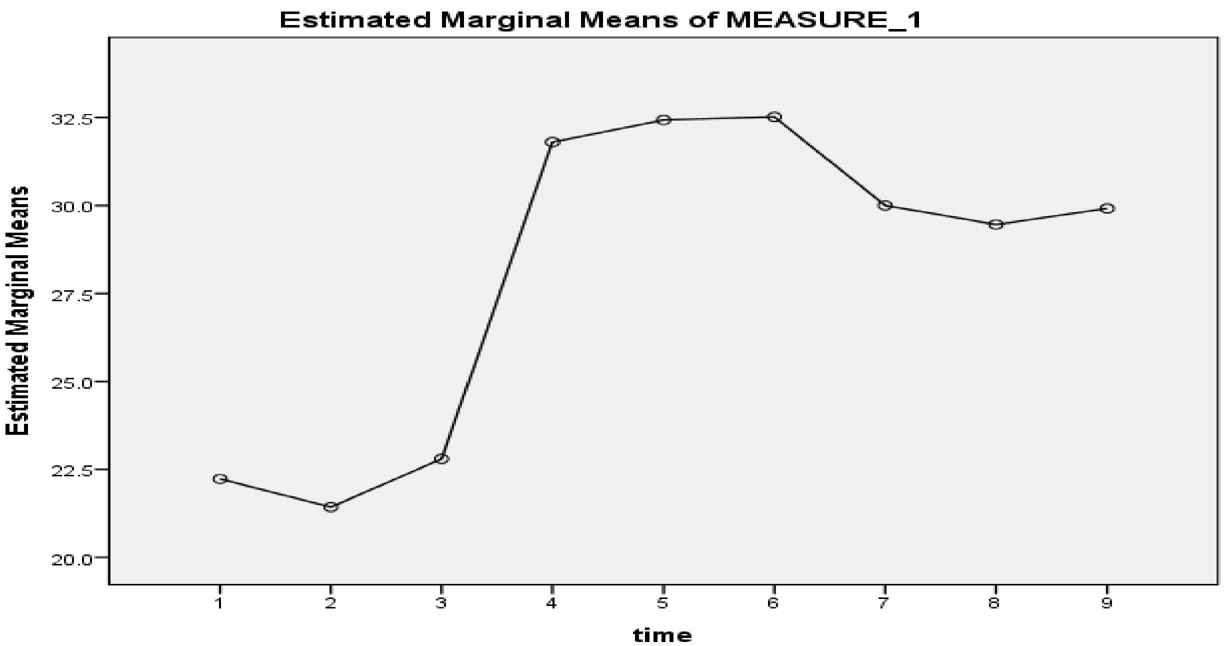

Figure 2: ***Effect of (CTBI) on Students' Critical Thinking Skills during Three Phases of Intervention***

The scores achieved by grade VIII students on critical thinking skills in the three phases of the study are shown in Figure 2. The SCORRAL baseline phase revealed that students' scores consistently scored low and were found to be stable with no critical thinking abilities before the intervention. In the intervention, test scores improved in a noticeable and upward trend, which indicates the positive impact of the critical thinking-based teaching strategies. However, scores during the withdrawal phase declined and plateaued at a level lower than that of the intervention phase but much higher than those recorded during the baseline phase. This indicates that while the removal of the intervention resulted in some regression, the students retained part of the skills acquired during the treatment.

Table 7: ***Effect of (CTBI) on Students' Critical Thinking Skills Regarding Concept of Polynomials during Three Phases of Treatment***

Phase	Observation	Mean (M)	Standard Deviation (SD)	Degrees of Freedom (df)	F-Value	Wilks' Lambda	p-Value	Partial Eta Squared
Baseline	Observation 1	22.23	1.43	14, 47	706.77	0.007	0	0.954
	Observation 2	20.97	1.01					
	Observation 3	22.23	1.43					
	Observation 4	21.63	1.16					
	Observation 5	23.54	1.22					
Intervention	Observation 1	31.8	1.23					
	Observation 2	33.97	1.465					
	Observation 3	31.8	1.232					
	Observation 4	33.97	1.465					
	Observation 5	36.31	2.336					
Withdrawal	Observation 1	30	1					
	Observation 2	30	1.2					
	Observation 3	30	1.1					
	Observation 4	30	1.11					
	Observation 5	30	0.0932					

***p < .001

The variation in ATCTS scores of students in the area of polynomials over five different measurement points is presented in Table 4.3.2. The analyses of the effect of the intervention on the CTS of grade VIII students in Mathematics were done by a one-way repeated measures ANOVA. The results revealed a significant difference in the mean critical thinking test scores over the ten weeks, **$F(14, 476) = 706.774$, $p = .000$, $\eta p^2 = .954$**. Further, the Bonferroni post hoc test indicated that the mean scores of the observations in weeks 3, 4, 5, 6, 7, and 8 differ significantly with respect to each other. This means that there is a clear effect from the intervention and consistent improvements from the five measurements in the experiment. These findings, however, provide a strong indication that our intervention was indeed effective at boosting students' ability to think critically about polynomials.

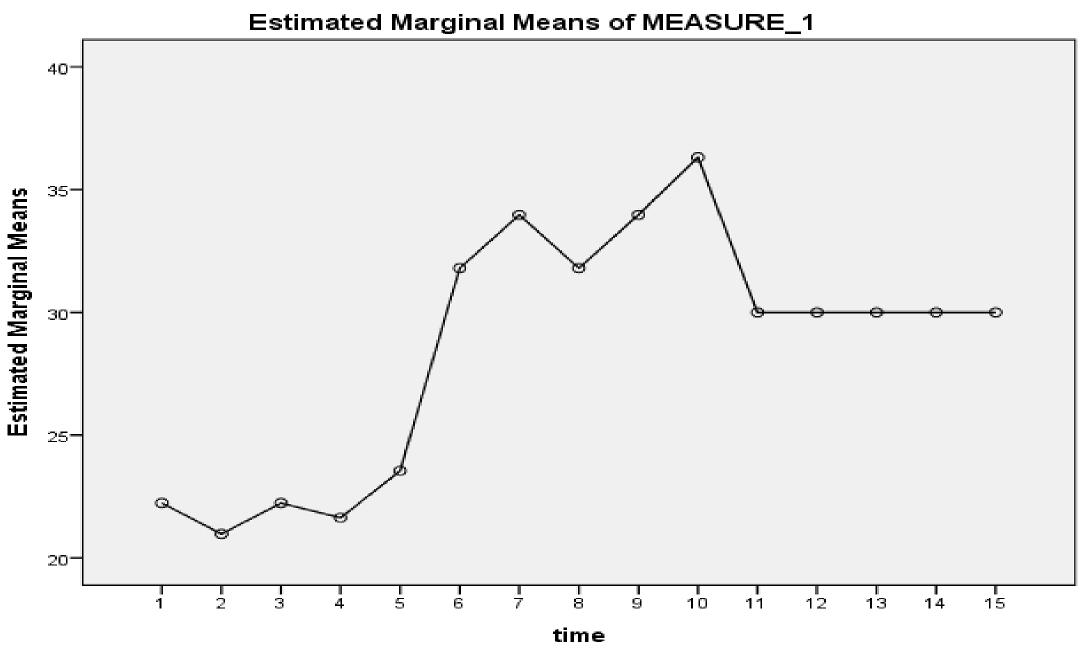

Figure 3: ***Effect of (CTBI) on Students' Critical Thinking Skills Regarding Concept of Polynomials during Three Phases of Intervention***

A progression of mean test scores in critical thinking skills of grade VIII students in Mathematics over three phases of the study is shown in Figure 3. The baseline phase scores were low and stable and not above the constant trend, which revealed an original lack of critical thinking capability in students before the intervention. In the intervention, we noted a significant improvement in test scores, a pattern of increasing scores which evidenced the positive influence of the teaching strategies essentially intended to enhance critical thinking. However, in the withdrawal phase, test scores dropped. This decrease in scores, however, was followed by stabilization of the scores at a level above those recorded during the baseline phase, indicating some retention of the critical thinking skills achieved during the intervention.

Table 8: ***Effect of (CTBI) on Students' Critical Thinking Regarding Concept of Factorization During Three Phases of Intervention***

Phase	Observation	Mean (M)	Standard Deviation (SD)	Degrees of Freedom (df)	F-Value	Wilks' Lambda	p-Value	Partial Eta Squared
Baseline	Observation 1	21.43	1.63	14, 47	718.2	0.008	0	0.955
	Observation 2	19.74	2.28					
	Observation 3	21.43	1.63					
	Observation 4	22.74	1.37					
	Observation 5	22.57	1.33					
Intervention	Observation 1	32.43	1.24					
	Observation 2	35.11	1.13					
	Observation 3	32.43	1.24					
	Observation 4	35.11	1.13					
	Observation 5	36.49	1.83					
Withdrawal	Observation 1	29.46	0.657					
	Observation 2	29.46	0.657					
	Observation 3	30.63	1.26					
	Observation 4	28.06	0.938					
	Observation 5	29.46	0.65					

***$p < .001$

The variation in ATCTS scores of grade VIII students in the factorization area across five distinct measurement points is shown in Table 4.3.3. A one-way repeated measure ANOVA was run to evaluate the impact of the intervention on students' Mathematics Critical Thinking Skills (CTS). The results revealed a significant difference in the mean scores over the ten weeks, **$F(14, 476) = 718.208$, $p = .000$, $\eta p^2 = .955$**. The Bonferroni post hoc test further indicated that the mean pair-wise scores for observations recorded in weeks 3, 4, 5, 6, 7, and 8 were significantly different from one another. These findings suggest that the intervention led to notable improvements in critical thinking skills, as evidenced by the significant differences in test scores across the five data points during the experiment.

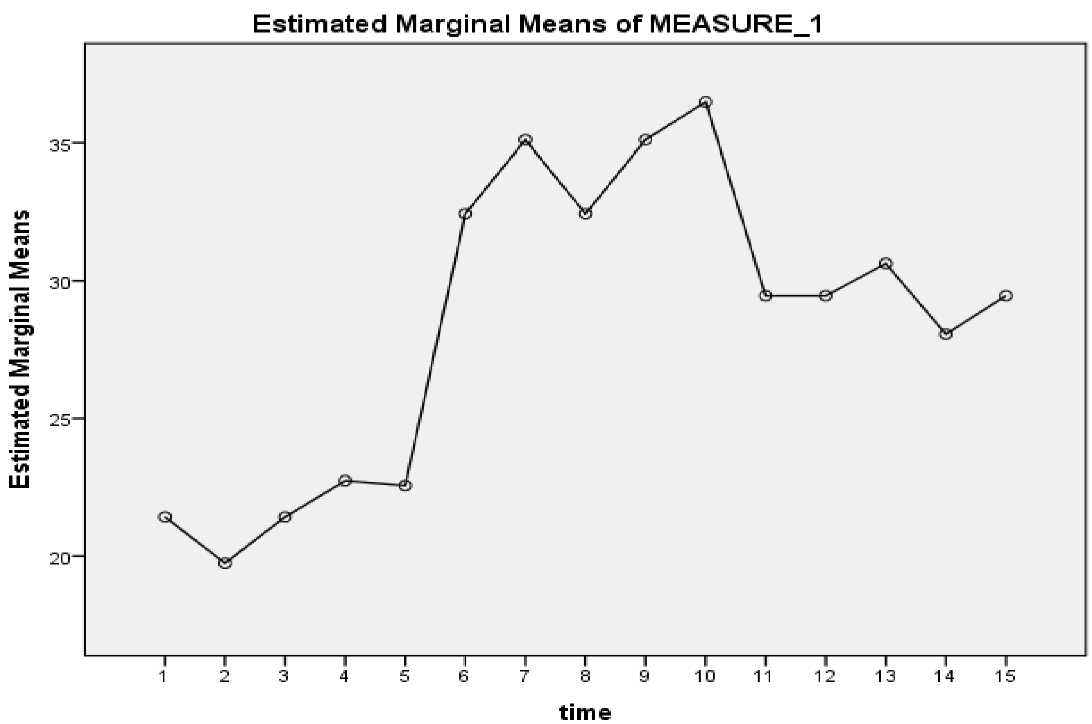

Figure 4: ***Effect of CTBI on Concept of Factorization***

Figure 4 illustrates the progression of mean test scores in critical thinking skills among grade VIII students in Mathematics. During the baseline phase, the scores were consistently low and followed a stable trend, reflecting minimal development in critical thinking skills prior to the intervention. In the intervention phase, there was a noticeable improvement in test scores, showing a clear upward trend, which indicates the positive impact of the applied teaching strategies. However, during the withdrawal phase, a decline in test scores was observed. Despite this decrease, the scores stabilized at a level that remained higher than those recorded in the baseline phase, suggesting partial retention of the skills developed during the intervention.

Table 9: ***Effect of (CTBI) on Students' Critical Thinking Skills Regarding Concept of Factorization during Three Phases of Intervention***

Phase	Observation	Mean (M)	Standard Deviation (SD)	Degrees of Freedom (df)	F-Value	Wilks' Lambda	p-Value	Partial Eta Squared
Baseline	Observation 1	22.8	1.32	14, 47	462.38	0.016	0	0.932
	Observation 2	21.94	1.39					
	Observation 3	22.8	1.32					
	Observation 4	22.69	1.43					
	Observation 5	23.71	1.07					
Intervention	Observation 1	32.51	1.17					
	Observation 2	34.69	2.32					
	Observation 3	32.51	1.17					
	Observation 4	34.69	2.32					
	Observation 5	35.66	2.57					
Withdrawal	Observation 1	29.91	0.28					
	Observation 2	29.91	0.28					
	Observation 3	31.03	0.98					
	Observation 4	28.2	1.18					
	Observation 5	29.91	0.28					

***$p < .001$

Table 9 presents the variations in ATCTS scores of grade VIII students in the area of factorization across five different measurement points. A one-way repeated measures ANOVA was conducted to evaluate how the intervention impacted students' critical thinking skills (CTS) in Mathematics. The results revealed a significant difference in the mean test scores over the ten-week period, **$F (14, 476) = 462.380$, $p = .000$, $\eta p^2 = .932$**. The Bonferroni post hoc test further indicated that the mean scores for observations taken in weeks 3, 4, 5, 6, 7, and 8 were significantly different from one another. These findings demonstrate that the intervention had a notable effect, resulting in significant improvements in test scores across the five data points during the experiment.

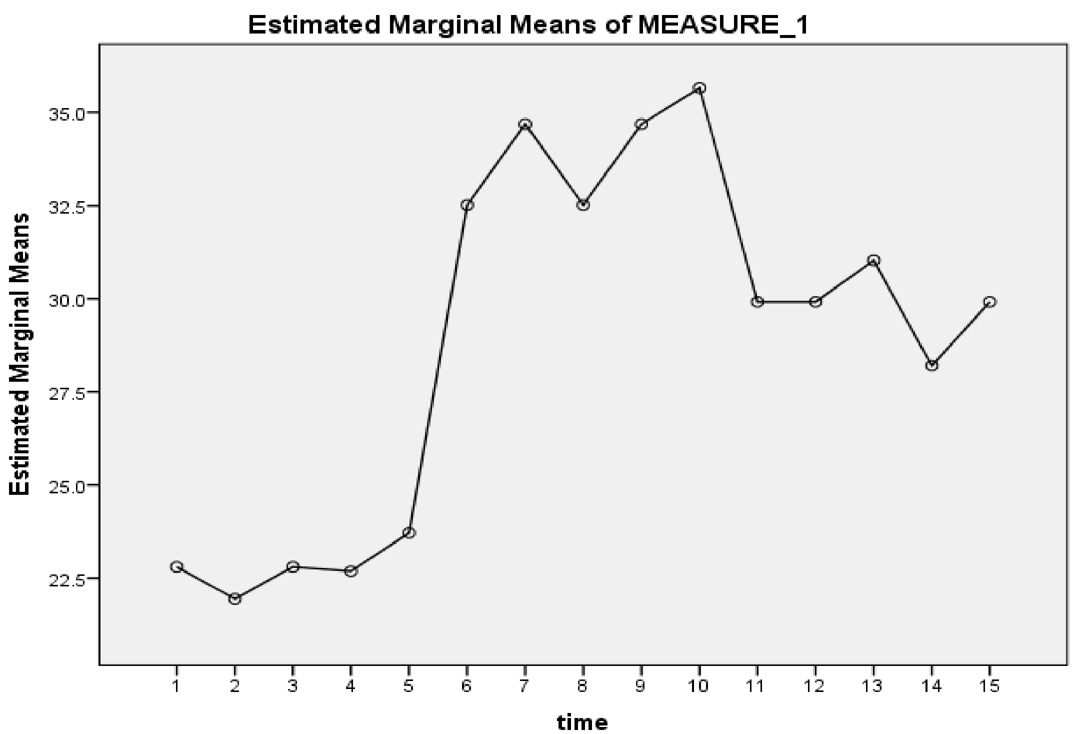

Figure 5: ***Effect of (CTBI) on Students' Critical Thinking Regarding Concept of Factorization during Three Phases of Intervention***

Figure 5 illustrates the average test scores in critical thinking skills among grade VIII students across three phases of the study. During the baseline phase, the scores were consistently low and followed a stable trend, indicating minimal development in critical thinking abilities prior to the intervention. In the intervention phase, the scores showed a significant increase, demonstrating a clear upward trend and highlighting the positive impact of the intervention on students' critical thinking skills. However, during the withdrawal phase, the scores declined and stabilized at a lower level compared to the intervention phase. Despite the decline, the mean test scores in the withdrawal phase remained higher than those recorded during the baseline phase, suggesting partial retention of the skills gained during the intervention.

Table 10: ***Effect of critical thinking skills on Misconceptions of students in Mathematics***

Test	Mean	SD	df	t-value	Sig(2-tailed)
Pre-Test	15.80	3.69	34	-15.041	.000
Post-Test	25.80	1.56			

Note: n=35, P=.005

Table 10 presents the paired t-test results evaluating the effectiveness of a critical thinking skills intervention in addressing students' misconceptions in mathematics. The **p-value (.000)** indicates a statistically significant impact of the intervention, demonstrating its success in reducing misconceptions related to mathematics learning. The data clearly highlights that the intervention was effective, leading to a meaningful reduction in students' misunderstandings. These findings confirm that the applied treatment significantly contributed to improving conceptual clarity in mathematics.

Table 11: ***Effect of critical thinking skills on Misconceptions of students in the Concept of Patterns***

Test	Mean	SD	df	t-value	Sig(2-tailed)
Pre-Test	4.7429	1.88403	34	-10.516	.000
Post-Test	7.9714	1.01419			

Table 11 presents the paired t-test results assessing the impact of a critical thinking skills intervention on addressing students' misconceptions in the concept of patterns. The **p-value (.000)** confirms a statistically significant effect of the intervention, demonstrating its effectiveness in reducing misconceptions related to learning mathematics. The findings from the table indicate that the intervention successfully improved students' understanding of patterns, highlighting its overall effectiveness in addressing and correcting misconceptions.

Table 12: ***Effect of critical thinking skills on Misconceptions of students in the Concept of Polynomials***

Test	Mean	SD	df	t-value	Sig(2-tailed)
PreTest	4.0286	1.38236	34	-12.158	.000
PostTest	7.2000	.79705			

Table 12 provides the paired t-test results evaluating the effectiveness of a critical thinking skills intervention in addressing students' misconceptions related to the concept of polynomials. The **p-value (.000)** indicates a statistically significant impact, confirming the intervention's success in reducing misconceptions in this area of mathematics. The data in the table demonstrates that the treatment was highly effective, leading to a substantial improvement in students' understanding of polynomials and successfully addressing their misconceptions.

Table 13: Effect of critical thinking skills on Misconceptions of students in the Concept of Fractions

Test	Mean	SD	df	t-value	Sig(2-tailed)
Pre-Test	3.8857	1.51019	34	-7.840	.000
Post Test	6.1714	.85700			

Table 13 presents the paired t-test results assessing the impact of a critical thinking skills intervention on students' misconceptions regarding the concept of fractions. The **p-value (.000)** demonstrates a statistically significant effect, indicating that the intervention successfully reduced misconceptions in this area of mathematics. The data reveals that the treatment effectively enhanced students' understanding of fractions, addressing and eliminating their misconceptions about the concept through targeted intervention.

Table 14: ***Effect of critical thinking skills on Misconceptions of students in the Concept of Linear Equations***

Test	Mean	SD	df	t-value	Sig(2-tailed)
Pre-Test	3.1429	1.47813	34	-8.695	.000
Post-Test	5.3714	.73106			

Table 14 presents the results of a paired t-test evaluating the impact of a critical thinking skills intervention on addressing students' misconceptions about linear equations. The **p-value (.000)** indicates a statistically significant effect, demonstrating the effectiveness of the intervention in reducing misconceptions related to this mathematical concept. The data clearly shows that the intervention significantly improved students' understanding of linear equations, effectively eliminating their misconceptions through targeted teaching strategies.

Conclusion

The study investigated elementary students' critical thinking (CTS) levels in mathematics, focusing on skills across patterns, fractions, polynomials, and linear equations. Most students showed basic skills in identification and interpretation but struggled with advanced skills, especially in fractions and linear equations. Misconceptions were common, including errors in pattern recognition, understanding algebraic rules, and solving equations. Results showed CTS interventions improved understanding and reduced misconceptions in areas like fractions and polynomials, rejecting the hypothesis that CTS had no effect on misconceptions. Statistical tests confirmed significant CTS improvement over phases.

Recommendations

For teachers, focus on CTS-based learning for challenging topics and use CTS modules to address misconceptions. For students, engaging in CTS activities is advised to enhance grades and understanding. Future research should explore CTS impacts at higher education levels and with cognitive-focused testing methods.

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