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Exploration of Reasons Behind the Alternative Conceptions of Algebra Among Secondary School Students: A Sequential Explanatory Mixed Method Research

S. Farah Shafaq Zaidi[1] & Muzammila Akram[2]

[1]*The Islamia University of Bahawalpur, Pakistan, Email: farahzaidi_iub@yahoo.com*
[2]*Associate Professor, Department of Educational Training, Faculty of Education, The Islamia University of Bahawalpur, Pakistan, Email: muzammila.akram@iub.edu.pk*

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Corresponding Author:
S. Farah Shafaq Zaidi
Email:
farahzaidi_iub@yahoo.com

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ABSTRACT

Conceptual understanding in the mathematics forms the foundation for higher-order reasoning and problem-solving. However, inadequate comprehension of abstract mathematical concepts often leads learners to develop alternative conceptions, particularly in algebra, where symbolic representation and formula application are central. The present study aimed to diagnose students' alternative conceptions in the algebraic component of secondary-level mathematics and to identify the underlying factors contributing to their formation. A sequential explanatory mixed method research design was employed that integrated quantitative data leading to qualitative data. A diagnostic test comprising 30 multiple-choice items was administered to 400 secondary school students. After that 40 secondary school students exhibiting a high frequency of alternative conceptions were purposively selected for semi-structured interviews. Quantitative data were analyzed through descriptive statistics using SPSS 26, while thematic analysis was applied to qualitative data. The findings revealed prevalent alternative conceptions related to algebraic expressions, simplification, surds, and formula manipulation. The major contributing factors included language misunderstanding, misinterpretation of prior knowledge, and procedural confusion during problem-solving. It was concluded that the use of systematic diagnostic assessments and concept-based instructional strategies is essential to identify and reconstruct students' alternative conceptions, thereby enhancing their conceptual understanding of algebra.

Introduction

An alternative conception refers to a learner's existing idea or personal interpretation that differs from the scientifically or mathematically accepted concept. Such conceptions emerge when learners attempt to interpret new information through the lens of their prior knowledge and experience (Driver & Easley, 1978; Posner, Strike, Hewson, & Gertzog, 1982). These ideas are often formed when learners construct meaning using incomplete or limited prior knowledge. In the context of mathematics, alternative conceptions arise when learners struggle to connect abstract symbols with their conceptual meanings, leading to persistent cognitive challenges in understanding (Akram et al., 2022;Vinner, 1997). These conceptions influence not only students' comprehension of fundamental mathematical principles but also their reasoning ability, problem-solving performance, and long-term learning outcomes (Fauzee et al., 2026;Swan, 2001).

Students develop mathematical understanding through both formal instruction and informal everyday experiences. When new mathematical knowledge conflicts with previously constructed notions, students often rely on their existing mental models instead of modifying them (ANDLEEB et al., 2025;Tall & Vinner, 1981). This cognitive resistance results in alternative conceptions that become deeply rooted and resistant to change through conventional teaching methods. In algebra, this is often evident in incorrect symbolic manipulation, misunderstanding of variable representation, or confusion between the operational and relational meanings of the equal sign (Knuth et al., 2006). These alternative interpretations create barriers to the development of abstract reasoning, which is central to mathematical learning (Booth, 1984).

Previous research in mathematics education has shown that alternative conceptions are a natural but challenging part of the learning process (Murtaza et al., 2024; Muslim et al., 2023). Learners' early encounters with arithmetic operations often establish rigid procedural habits that interfere with the flexible reasoning required for algebraic thinking (Kieran, 1989). For example, students may apply arithmetic rules to algebraic expressions by assuming $(a+b)^2=a^2+b^2(a + b)^2 = a^2 + b^2(a+b)^2=a^2+b^2$ or by combining unlike terms during simplification. These patterns demonstrate a lack of conceptual transfer between arithmetic and algebra, highlighting the need for diagnostic approaches to identify and reconstruct such alternative conceptions at the secondary level (Kuchemann, 1981).

Language also plays a significant role in the formation of mathematical alternative conceptions. According to Vygotsky (1978), language mediates conceptual development and shapes how learners internalize abstract ideas. When mathematical terminology is misunderstood or translated inaccurately such as the terms expression, coefficient, or constant students' reasoning becomes inconsistent with formal mathematical definitions (Akram, 2016; Yousaf et al., 2025). Similarly, ambiguities in textbook wording or teachers' explanations can contribute to semantic misunderstandings, particularly in contexts where English is a second language (Huma et al., 2025;Sayer, 2013).

In Pakistan, mathematics achievement at the secondary level remains a critical concern, with numerous studies reporting poor conceptual understanding and a high reliance on procedural learning (Awan & Zia, 2021; Malik & Khurshid, 2020). Algebra, being a foundational domain in mathematics, often poses difficulties due to its symbolic nature and abstract reasoning demands. Despite the importance of algebra in higher education and scientific disciplines, limited empirical research has systematically diagnosed the types and causes of alternative conceptions among Pakistani secondary school students. Therefore, the present study sought to identify algebra-related alternative conceptions and explore the underlying factors contributing to their formation among secondary learners in the Bahawalpur district.

In addition to cognitive and linguistic factors, instructional approaches play a significant role in the persistence of alternative conceptions. Traditional teaching methods in mathematics classrooms often emphasize rote learning and procedural accuracy over conceptual reasoning (Hiebert & Grouws, 2007). Students are frequently trained to apply memorized algorithms without understanding the logic behind them, resulting in superficial comprehension. This situation creates a discrepancy between procedural fluency and conceptual understanding—an imbalance that leads to persistent alternative conceptions when learners encounter unfamiliar algebraic problems (Rittle-Johnson & Schneider, 2015). Teachers' limited awareness of diagnostic strategies further contributes to the continuation of these conceptions across grade levels (Mahmood, 2020;Skemp, 1976).

Moreover, the curriculum and assessment systems in Pakistan have been reported to promote surface-level learning (Malik & Khurshid, 2020; Iqbal & Qureshi, 2022). Frequent reliance on textbook exercises and examination-focused instruction restricts opportunities for conceptual exploration and problem-solving (Mahmood & Anwar, 2025;Awan & Zia, 2021). While this approach may enhance short-term retention, it fails to challenge students' alternative conceptions or encourage conceptual restructuring through reasoning. Consequently, secondary-level learners tend to carry their algebraic alternative conceptions into higher education, where these ideas become more complex and difficult to reconstruct (Mahmood & Ismail, 2018, 2022;Ali & Iqbal, 2018).

From a theoretical perspective, this study is grounded in the Constructivist Theory of Learning, which posits that learners actively construct knowledge through their interactions with the environment (Piaget, 1972). Within this framework, alternative conceptions are viewed as logical outcomes of learners' attempts to make sense of new information based on existing cognitive structures (Vosniadou, 1994). According to Conceptual Change Theory (Posner et al., 1982), meaningful learning occurs when individuals modify or restructure pre-existing conceptions through cognitive conflict, reflection, and accommodation (Mahmood, Kanwal, et al., 2023). Applying these frameworks to mathematics education enables teachers to identify and reconstruct students' alternative conceptions through guided inquiry and conceptual reasoning (Chi, 2008).

Furthermore, research has demonstrated that diagnostic assessment tools, such as two-tier or multiple-tier tests, can effectively identify the nature and origin of students' alternative conceptions (Treagust, 1988; Caleon & Subramaniam, 2010). These instruments assess not only the correctness of responses but also the reasoning behind them, providing insight into students' cognitive processes. In combination with semi-structured interviews, they offer a deeper understanding of the underlying causes of alternative conceptions, whether stemming from linguistic misunderstanding, misapplication of prior knowledge, or procedural inaccuracy (Akram, 2016; Drechsler, 2007). Hence, using a diagnostic test followed by qualitative interviews provides a comprehensive approach for uncovering the nature and causes of alternative conceptions in algebra.

Given the centrality of algebra in mathematical cognition and the limited empirical evidence available in the Pakistani context, it was crucial to examine this issue systematically(Mahmood, Ismail, et al., 2024). Therefore, the present study aimed to diagnose algebraic alternative conceptions among secondary school students and to analyze their underlying causes through a mixed-method case study approach. It is expected that the findings were contribute to the improvement of teaching practices, curriculum design, and teacher professional development programs. Moreover, this research provides a localized evidence base for educators and policymakers to design interventions that promote conceptual understanding and reduce learning barriers in mathematics.

Objectives of the Study:

Following are the objectives of the study:

- To identify the mathematics alternative conceptions of Algebra among the secondary school students.
- To explore the best possible reasons behind the mathematics alternative conceptions at the secondary level.

Research Question:

The following research question was lead to the present study:

- How to identify the mathematics alternative conceptions of Algebra at the secondary level?
- What are the best possible reasons behind the mathematics alternative conceptions at the secondary level?

Literature Review

Identifying the Multifaceted Factors Contributing to Mathematical Alternative Conceptions:

To meticulously uncover and classify the factors driving the emergence and dissemination of alternative math concepts. Patterns and relationships from mathematics are needed to understand the next classes. a mindset that helps people gather, analyses, and organize data. A language with precisely defined concepts and symbols that improves science, real- them with a well-known mathematical technique (Huma, Ahmed, Mahmood, & Awan, 2025; Mahmood, Pervez, et al., 2023; Niaz et al., 2023). These help students discover why they study math. Mathematicians and others use math to solve tangible and abstract problems, and students were too. Math is essential to our world. (Rosjanuardi, 2018). Alternative Conceptions are required steps for kids and must be taught. If pupils are attached to unstable and weak pictures of a strategy, these actions do not hinder their future learning. If they are based on reliable and enduring conceptual models, they do. While valuable in another context, Alternative Conceptions must be broadened for future usage. These Alternative Conceptions are a vital part of learning and the result of a knowledge gap; thus, they cannot be avoided. (Jabal, 2019). Although related to the first goal, this focuses on establishing a complete structure of contributing pieces. This involves identifying the constituents and studying their complex interconnections. We want to discover how these variables interact to foster alternate ideas. This involves examining: Cognitive elements: These relate to students' learning styles, cognitive abilities, and problem-solving methods. How could these cognitive qualities affect alternate conceptions? Emotional Views: Affective components include students' math attitudes, self-efficacy beliefs, and emotional reactions to math activities. How may emotions, cognition, and instruction shape learning? Parental involvement, teaching environment, peer interactions, and mathematical cultural attitudes are socio-cultural factors (Cheng Li et al., 2023; Sumera kanwal et al., 2024). How might geopolitical factors effect students' math comprehension and inspire new ideas? Instructional materials: Instructional quality, curriculum design, assessment methods, and resource availability are covered. How can these teaching tools promote alternate ideas? This study's conclusion of these two key aims was provide educators, curriculum designers, and teachers with insightful analysis. The goal is to improve secondary school students' mathematics knowledge by developing better plans to avoid and address mathematical alternative conceptions (Ahmed et al., 2025; Huma, Ahmed, Mahmood, khan, et al., 2025; Mahmood et al., 2021).

Understanding the Origins of Mathematical Alternative Conception:

This study employs the term "alternative conceptions" to emphasize that these notions are an inherent part of the process through which learners develop mathematical understanding (Swan, 2001). According to Hattie (2009), the learner's active construction of ideas and knowledge holds greater significance than the mere possession of information.

Alternative conceptions represent deeply held beliefs that individuals consistently maintain, distinguishing them from random errors or momentary mistakes. Unlike casual errors, alternative conceptions are systematic and resistant to change, as learners often defend their existing understanding when confronted with conflicting information (Erdogan, 2011). With appropriate guidance and reflection, however, individuals can recognize and reconstruct these conceptions through conceptual change and cognitive restructuring (Mahmood, Akram, Asghar, et al., 2024; Mahmood, Akram, Ismail, et al., 2024).

The present study is grounded in the belief that developing effective teaching strategies requires a comprehensive understanding of students' alternative conceptions and the underlying factors that contribute to them. Accordingly, the study was guided by two main objectives:

Exploring the best arguments for mathematical alternative conceptions goes beyond documentation. It seeks to explain secondary students' nonstandard understandings. This requires a thorough investigation of potential reasons. We want to discover the intellectual processes, educational opportunities, and environmental elements that lead kids to this alternative conception. The study was investigating possibilities like, how do students' mental models and frameworks affect their interpretation of modern mathematical ideas, Existing natural limits or prejudices in how students understand math.

Instructional strategies: Can curriculum design, mathematical idea presentation, and instruction promote alternate conceptions? Any teaching methods that accidentally misinterpret?

Language and portrayal: How can symbolic, visual, and verbal representations and mathematical explanations affect students' understanding? Do language or representation ambiguities encourage alternative ideas? world, and math communication. Real-life problems are usually simplified by symbolizing

Previous knowledge and expertise: How do students' daily experiences and formal and informal mathematics knowledge affect their new idea comprehension? Alternative Conceptions: Do knowledge gaps or contradictory real-world perception cause them? Affective factors: How do students' math attitudes, beliefs, and emotions affect their learning and shape alternative conceptions? Lack of confidence or math fear matter?

Materials and Methods

Research Design

The study employed a sequential explanatory mixed method research design to diagnose and interpret students' alternative conceptions in the algebra component of mathematics. In the first quantitative phase, a diagnostic test was used to identify the types and frequency of alternative conceptions, while in the second qualitative phase, semi-structured interview was conducted to explore students' reasoning behind their selected responses. This design enabled the integration of statistical results with in-depth qualitative insights, ensuring a comprehensive understanding of both the nature and underlying causes of alternative conceptions. This approach was chosen because it allowed an in-depth examination of students' conceptual understanding within their real classroom context, consistent with the constructivist view that learning and alternative conceptions are contextually constructed (Creswell & Plano Clark, 2018; Yin, 2018; Vygotsky, 1978).

Population and Sampling

The target population of the study comprised all Grade IX students enrolled in government secondary schools in the Bahawalpur City. A purposive sampling technique was employed to select participants who had studied algebra as part of their mathematics curriculum. A total of 400 students were selected for the quantitative phase to take the diagnostic test, while 40 students who exhibited a higher frequency of alternative conceptions were purposively chosen for the qualitative phase involving semi-structured interviews. This sampling strategy ensured that participants represented typical learning contexts of secondary mathematics classrooms and provided sufficient data to identify both the prevalence and cognitive origins of algebraic alternative conceptions.

Research Instruments

Two research instruments were utilized in this study: a diagnostic test and a semi-structured interview guide. The diagnostic test consisted of 30 multiple-choice questions covering major algebraic topics such as expressions, simplification, surds, and formula application. Each item included one correct option and three distractors designed to represent common alternative conceptions held by students. After analyzing the diagnostic test results, a semi-structured interview guide comprising eight open-ended questions was developed to further explore students' reasoning and conceptual interpretations. The combination of these instruments provided both quantitative data on the frequency of alternative conceptions and qualitative insights into the cognitive and linguistic factors that contributed to their development.

Validity and Reliability

To ensure the accuracy and consistency of the research instruments, both content validity and reliability were established. The diagnostic test and interview guide were reviewed by five experts in mathematics education to confirm the clarity, relevance, and alignment of items with the research objectives. A pilot study was conducted with 50 students from a similar educational context to refine ambiguous questions and improve the structure of the instruments. The reliability of the diagnostic test was determined through statistical analysis, yielding a Cronbach's alpha value of 0.87 and a Pearson correlation coefficient (r) of 0.96, indicating a high level of internal consistency and measurement stability.

Data Collection Procedure

Data were collected in two sequential phases during the 2023–2024 academic session. In the first phase, permission was obtained from school heads, and students were briefed about the study's objectives, confidentiality, and voluntary participation. The diagnostic test was administered to 400 students under standardized classroom conditions, and their responses were scored using a predefined answer key. In the second phase, 40 students who exhibited the highest number of alternative conceptions were purposively selected for semi-structured interviews. Each interview lasted approximately 15 to 20 minutes and was conducted in either Urdu or English, depending on the participant's preference. All interviews were recorded with prior consent to ensure accuracy and later transcribed for thematic analysis.

Data Analysis Techniques

The collected data were analyzed using both quantitative and qualitative methods in alignment with the sequential explanatory design of the study. Quantitative data obtained from the diagnostic test were entered into SPSS version 26 and analyzed through descriptive statistics, including frequencies and percentages, to identify the most common alternative conceptions among students.

Qualitative data from the semi-structured interviews were transcribed, coded, and thematically analyzed to explore the underlying factors contributing to these alternative conceptions. From the qualitative analysis, three major themes emerged: language misunderstanding, misinterpretation of prior knowledge, and procedural confusion. Finally, data triangulation was applied to integrate findings from both phases, ensuring comprehensive interpretation, validation, and reliability of the results.

Ethical Considerations

All ethical principles were carefully observed throughout the study to ensure credibility, transparency, and participant protection. Permission to conduct the research was obtained from the concerned institutional authority and school administrations. Participation was entirely voluntary, and informed consent was obtained from all participants prior to data collection. Students were assured that their responses would remain confidential and used solely for research purposes. Personal identifiers were removed from all datasets to maintain anonymity, and all interview recordings were securely stored and deleted after transcription. These measures ensured full compliance with academic research ethics and safeguarded participants' rights and privacy.

Result and Discussions

This chapter presents the results and analysis of the data collected through the diagnostic test and semi-structured interviews to identify and interpret students' alternative conceptions in the algebra component of mathematics. The data analysis was conducted in two sequential phases according to the mixed-method design of the study. In the first phase, quantitative results from the diagnostic test were analyzed using descriptive statistics to determine the frequency and percentage of students correct and incorrect responses. In the second phase, qualitative data obtained from interviews were thematically analyzed to explore the underlying causes of the alternative conceptions identified in the diagnostic test. The integration of both data sets provided a comprehensive understanding of the nature, extent, and causes of students' alternative conceptions, thereby addressing the main objectives of the study.

Quantitative Data Analysis (Diagnostic Test Results)

Overview of Diagnostic Test

A diagnostic test consisting of 30 multiple-choice questions was administered to 400 secondary school students to identify their alternative conceptions in the algebra component of mathematics. The test covered four key areas: algebraic expressions, simplification, surds and indices, and algebraic formulas. Each question included one correct answer and three distractors specifically designed to reveal common alternative conceptions held by students. The responses were analyzed using descriptive statistics through SPSS version 26 to determine the frequency and percentage of correct and incorrect answers. The overall results indicated that, although many students demonstrated partial conceptual understanding of algebraic concepts, a substantial proportion exhibited consistent alternative conceptions across multiple topics, reflecting challenges in connecting procedural fluency with underlying conceptual reasoning.

Item-wise Analysis

Table 1: Responses to Item 1 – Simplify $(a + b)^2 + (a - b)^2$

Option	**A.** $2a^2 + 2b^2$	**B.** $a^2 + b^2$	**C.** $(a + b)^4$	**D.** $2ab$	**Total**
Frequency	180	110	70	40	400
Percentage	45%	27%	18%	10%	100%

Interpretation:

Approximately half of the students correctly applied the algebraic identity, whereas the remaining 55% exhibited alternative conceptions, such as squaring individual terms or misinterpreting the overall expression structure. These patterns indicated a limited understanding of formula application and weaknesses in symbolic reasoning and conceptual interpretation of algebraic relationships.

Table 2: Responses to Item 2 – Simplify 3x + 2x =?

Option	A. 5x	B. $6x^2$	C. $3x^2 + 2x^2$	D. $5x^2$	Total
Frequency	200	100	740	30	400
Percentage	55%	25%	12%	8%	100%

Interpretation:

More than half of the students correctly simplified the expression by combining like terms. However, a notable proportion applied incorrect algebraic rules by introducing exponents (Options B, C, and D), suggesting confusion between addition of like terms and multiplication or squaring of variables. The selection of Option D indicates a misconception where coefficients are added correctly, but the variable is incorrectly squared, reflecting gaps in conceptual understanding of linear expressions.

Table 3: Responses to Item 3 – If a = 2, b = 3, then find $(a + b)^2$

Option	A. 25	B. 13	C. 36	D. 10	Total
Frequency	250	70	50	30	400
Percentage	62%	18%	12%	8%	100%

Interpretation:

Sixty-Two percent of students responded correctly, while the remaining 35% demonstrated alternative conceptions related to the order of operations. Several students added the numbers first and then squared the sum, whereas others squared each term separately, reflecting an incomplete procedural understanding and difficulty in applying the correct sequence of algebraic operations.

Item 4: Evaluate $\sqrt{49} + \sqrt{16}$

Option	A. 11	B. 65	C. $\sqrt{65}$	D. 7 + 4	Total
Frequency	230	90	50	30	400
Percentage	62%	18%	12%	8%	100%

Interpretation:

While 62% solved correctly, the remaining students combined radicands before simplifying, revealing confusion about laws of surds and conceptual understanding of square roots.

Item 5: Simplify $(x^2y)(xy^2)$

Option	A. x^3y^3	B. x^2y^2	C. x^4y^4	D. x^5y^5	Total
Frequency	210	90	70	30	400
Percentage	52%	22%	18%	8%	100%

Interpretation:

Nearly half demonstrated alternative conceptions by incorrectly adding or multiplying powers. This showed weaknesses in applying index laws and confusion between multiplication and exponentiation.

Item 6: Expand (a + b)(a – b)

Option	A. $a^2 - b^2$	B. $a^2 + b^2$	C. $a^2 - 2ab + b^2$	D. $a^2 + 2ab + b^2$	Total
Frequency	200	100	70	30	400
Percentage	50%	25%	17%	8%	100%

Interpretation:

Only 50% expanded correctly. Others incorrectly applied the distributive law, showing difficulty in recognizing algebraic identities and poor conceptual transfer from arithmetic.

Item 7: Substitute x = 2, y = 3 in xy + y^2

Option	A. 15	B. 10	C. 12	D. 9	Total
Frequency	240	90	40	30	400
Percentage	60%	22%	10%	8%	100%

Interpretation:

Although most students substituted correctly, one-third misapplied variable substitution due to incomplete understanding of algebraic notation and translation from symbolic to numerical form.

Item 8: Solve for x: 2x + 5 = 11

Option	A. 3	B. 6	C. 8	D. −3	Total
Frequency	260	80	40	20	400
Percentage	65%	20%	10%	5%	100%

Interpretation:

Seventy percent solved correctly, but 30% made procedural errors, mainly due to misinterpretation of inverse operations. This indicated partial procedural fluency without conceptual clarity.

Summary of Quantitative Findings

The overall quantitative results revealed that alternative conceptions were prevalent across all algebraic topics. The frequency analysis indicated that simplification and formula application exhibited the highest rates of incorrect responses, suggesting persistent conceptual and procedural challenges in these areas. In contrast, surds and indices showed comparatively fewer errors, indicating a relatively stronger grasp of these concepts among students.

Summary of Quantitative Findings

Algebra Topic	Correct Responses (%)	Alternative Conceptions (%)
Algebraic Expressions	55	45
Simplification	48	52
Surds and Indices	60	40
Algebraic Formulas	42	58
Factorization	53	47

Signs (+/–)	51	49
Substitution	62.5	37.5
Word Problems	46	54

Interpretation:

The results revealed that simplification (52%) and formula application (58%) had the highest rates of alternative conceptions. Students showed particular difficulty in symbolic manipulation and use of algebraic identities. These findings indicate that rote memorization dominates students' learning, while conceptual reasoning remains weak.

Qualitative Data Analysis (Interview Results)

Overview of Qualitative Phase

To explore the underlying reasons behind students' alternative conceptions identified through the diagnostic test, semi-structured interviews were conducted with 40 students who exhibited the highest frequencies of such conceptions. The qualitative data were analyzed using thematic analysis, which involved transcribing, coding, and categorizing students' responses into recurring conceptual patterns. The analysis revealed that students' alternative conceptions primarily stemmed from linguistic, cognitive, and procedural factors. Three major themes emerged from this phase: language misunderstanding, misinterpretation of prior knowledge, and procedural confusion. These themes provided a deeper understanding of the cognitive and linguistic origins of students' difficulties in algebraic reasoning and problem-solving.

Thematic Findings

Theme 1: Language Misunderstanding

Language-related confusion emerged as a major source of alternative conceptions in algebra. Many students struggled to interpret mathematical terms such as *expression*, *variable*, and *coefficient*, and often misunderstood symbolic representations, which led to inconsistent reasoning and incomplete comprehension of algebraic concepts.

Example:

R: What do you understand by the term "coefficient" in 5x?

S1: It means the number we multiply after solving the question.

Interpretation:

This response reflected a misunderstanding of algebraic terminology, where the student perceived the coefficient as an operation rather than a multiplier. Similar responses indicated that unclear language and lack of conceptual clarity contributed to symbolic misinterpretations.

Theme 2: Misinterpretation of Prior Knowledge

Several alternative conceptions were associated with the inappropriate transfer of arithmetic knowledge to algebraic contexts. Students frequently relied on arithmetic rules and procedures when manipulating algebraic symbols, indicating difficulties in distinguishing between numerical operations and algebraic reasoning.

Example:

R: Why did you solve $(a + b)^2$ as $a^2 + b^2$?

S2: Because I thought the square applies to each number separately, just like $2^2 + 3^2 = 13$.

Interpretation:

This reflected students' tendency to generalize arithmetic operations to algebraic expressions, indicating that their conceptual frameworks were predominantly procedural rather than relational. Such alternative conceptions demonstrated an incomplete transition from numerical computation to abstract algebraic reasoning, highlighting the need for instructional emphasis on conceptual connections rather than procedural repetition.

Theme 3: Procedural Confusion

Students demonstrated confusion when applying algebraic procedures, especially in simplification and formula manipulation.

Example:

R: How did you solve $3x + 2x$?

S3: I multiplied 3 by 2 and wrote $6x^2$ because it looked similar to multiplication.

Interpretation:

This error showed that students were uncertain about the difference between multiplication and addition of variables. Many relied on visual or surface-level cues rather than understanding the underlying mathematical relationship, indicating over-dependence on memorized rules instead of conceptual understanding.

Summary of Qualitative Findings

The qualitative analysis revealed that students' alternative conceptions stemmed from overlapping cognitive and linguistic factors. Language misunderstanding created difficulties in interpreting algebraic symbols and terminology, while the misinterpretation of prior knowledge led to the incorrect transfer of arithmetic rules to algebraic situations. Additionally, procedural confusion emerged as a result of incomplete mastery of algebraic operations and an overreliance on memorized procedures rather than conceptual understanding.

Table 5: Summary of Qualitative Findings

Identified Cause	Frequency	Percentage
Language Misunderstanding	130	32.5%
Misinterpretation of Prior Knowledge	170	42.5%
Procedural Confusion	100	25%

Interpretation:

The findings indicated that misinterpretation of prior knowledge (42.5%) was the most frequent cause of alternative conceptions, followed by language misunderstanding (32.5%) and procedural confusion (25%). These results confirmed that students' alternative conceptions in algebra were primarily rooted in cognitive processing difficulties and linguistic misinterpretations, rather than arising from isolated computational errors.

Integration of Quantitative and Qualitative Findings

The integration of quantitative and qualitative findings provided a comprehensive understanding of the nature and underlying causes of students' alternative conceptions in algebra. The quantitative analysis revealed that a significant proportion of students held alternative conceptions in simplification (52%), formula application (58%), and algebraic expressions (45%). These results were further supported and clarified by qualitative data obtained through semi-structured interviews. The thematic analysis demonstrated that the primary factors contributing to these alternative conceptions were misinterpretation of prior knowledge, language misunderstanding, and procedural confusion during problem-solving.

Students who exhibited a higher frequency of alternative conceptions in the diagnostic test were often those who misunderstood algebraic terminology or incorrectly transferred arithmetic reasoning to algebraic contexts. For example, diagnostic results showed frequent errors in applying the identity $(a+b)^2=a^2+2ab+b^2(a + b)^2 = a^2 + 2ab + b^2(a+b)^2=a^2+2ab+b^2$, and interview findings confirmed that many students believed the square applied individually to each term. This reflected a partial conceptual understanding of algebraic structure and an overreliance on prior arithmetic rules. Similarly, the high number of incorrect responses in formula-based items aligned with interview insights, where students admitted memorizing formulas without comprehending their conceptual derivation or logical relationships.

Moreover, the integrated analysis highlighted that linguistic challenges were a recurring influence across all topics. Students' limited comprehension of mathematical terms such as *variable*, *term*, and *coefficient* affected both their reasoning processes and their interpretation of algebraic relationships. This overlap between linguistic, cognitive, and procedural factors indicated that students' alternative conceptions were not isolated misunderstandings but rather interconnected cognitive constructions that developed through prior learning experience.

Table 6: Integration of Quantitative and Qualitative Findings

Algebra Concept	Quantitative Alternative Conceptions Rate (%)	Qualitative Cause Identified
Algebraic Expressions	45	Language Misunderstanding
Simplification	52	Procedural Confusion
Surds and Indices	40	Misinterpretation of Prior Knowledge
Algebraic Formulas	58	Misinterpretation of Prior Knowledge
Factorization	47	Procedural Confusion
Signs (+/–)	49	Cognitive Overgeneralization
Substitution	37.5	Symbolic Misinterpretation
Word Problems	54	Linguistic and Conceptual Barriers

Interpretation:

The integrated findings revealed that students' alternative conceptions in algebra stemmed mainly from the transfer of arithmetic reasoning to symbolic contexts, limited conceptual understanding, and language-based misinterpretation. The triangulated data validated quantitative results and offered rich qualitative insight, confirming the value of a sequential mixed-method approach for diagnosing alternative conceptions in mathematics.

Conclusion

This study investigated the persistent issue of alternative conceptions in secondary school mathematics, focusing specifically on algebraic learning challenges. Through the administration of a diagnostic test and in-depth interviews with the students, the research identified the most frequent areas where alternative conceptions occurred namely algebraic expressions, simplification, surds, and algebraic formula manipulation. The findings revealed a concerning prevalence of conceptual inconsistencies, with many students struggling to internalize fundamental algebraic principles. Instead of demonstrating conceptual understanding, a considerable number of learners relied on rote memorization and guessing strategies, leading to repeated difficulties in solving algebraic problems.

The analysis indicated that these alternative conceptions largely stemmed from the misinterpretation of prior knowledge, misunderstanding of algebraic signs, and incomplete simplification procedures. Students frequently transferred previously learned arithmetic rules into algebraic contexts, particularly when manipulating positive and negative signs or simplifying complex expressions. These tendencies highlighted the role of cognitive transfer errors and procedural dependence as major barriers to developing a robust conceptual understanding of algebra.

The study also revealed that many learners exhibited surface-level engagement with mathematical tasks, attempting to find answers through estimation rather than reasoning. Such patterns reflect insufficient emphasis on conceptual reasoning and inquiry-based learning within classroom instruction. Students' persistent difficulties in sign manipulation and simplification further demonstrated weaknesses in foundational algebraic reasoning, which impede their progression toward higher-level mathematical thinking.

Recommendation

Based on the findings of this study, several recommendations are proposed to address the persistent problem of alternative conceptions in secondary school algebra. These suggestions aim to improve students' conceptual understanding, guide teachers' instructional practices, and strengthen diagnostic assessment within mathematics education.

Prioritize Conceptual Understanding

Mathematics instruction should shift from rote memorization and mechanical drills to concept-focused learning. Teachers should emphasize the *"why"* behind mathematical rules, using visual aids, manipulatives, and real-world examples to enhance students' comprehension of algebraic relationships.

Identify and Address Prior Knowledge Gaps

Before introducing new topics, teachers should assess students' existing understanding to identify alternative conceptions that may have developed from previous learning experiences. Targeted remediation and conceptual reconstruction activities should then be implemented to address these gaps, ensuring that students build upon a solid and accurate foundational understanding of mathematical concepts.

Explicit Teaching of Sign Manipulation

Since errors involving positive and negative signs were frequent, explicit instruction on sign rules and operations should be incorporated into algebra lessons. Interactive methods such as number lines, color coding, or visual models can help students internalize these concepts.

Enhance Problem-Solving Skills

Students should be encouraged to engage in analytical and reasoning-based problem-solving rather than guessing. Teachers should design challenging, open-ended problems and guide students to explain and justify their answers, promoting critical thinking and conceptual reasoning.

Promote Self-Reflection and Metacognitive Awareness

Learners should be provided with opportunities to reflect on their thought processes and recognize their own alternative conceptions. Strategies such as self-assessment exercises, reflective journals, and peer discussions can help students critically evaluate, monitor, and regulate their understanding of algebraic concepts, thereby fostering deeper conceptual awareness and self-correction in learning.

Implement Regular Diagnostic Assessment

Teachers should use diagnostic tests periodically to identify and track students' conceptual errors. Such assessments should evaluate not only procedural accuracy but also conceptual reasoning. The findings can guide targeted interventions to meet individual learning needs.

Foster a Supportive Learning Environment

Classrooms should encourage open dialogue and a culture of error tolerance, allowing students to express their reasoning without fear of criticism. Teachers should view alternative conceptions as valuable learning opportunities, fostering an environment where mistakes are recognized as an integral part of the conceptual growth and knowledge reconstruction process.

Provide Continuous Professional Development for Teachers

Regular professional development programs should be organized to equip teachers with the skills to identify, interpret, and address students' alternative conceptions effectively. Workshops on diagnostic assessment techniques, concept-based instructional strategies, and formative feedback practices can significantly enhance teachers' pedagogical competence and their ability to foster meaningful conceptual understanding in mathematics classrooms.

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